CHANGE OF HYDRODYNAMIC DRAG OF A SPHERE SET IN MOTION BY ELECTROMAGNETIC FORCES

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Many papers on the problems of MHD flow around bodies in external magnetic fields are surveyed in [1, 2]. A similar investigation for bodies with internal sources of electromagnetic fields is of interest. The electric and magnetic fields in the fluid are produced only in the neighborhood of the body around which the flow occurs, and what is particularly important, there is the possibility of controlling the magnitude and spatial distribution of body forces. Such internal sources may act as a motor, setting a solid body into translational motion relative to the fluid. In this case the effect of electromagnetic body forces (EBF) on the flow pattern and the magnitude of the hydrodynamic drag is particularly interesting.

The problem can be studied completely only by combined theoretical and experimental research; pure theoretical calculations using numerical methods at the present time are limited to small values of the Reynolds number Re. Calculations [3] showed that the hydrodynamic drag of a plate set in motion in its own plane by EBF is higher than the classical value. This increase is due to the acceleration of the fluid in the neighborhood of the plate, and in the example considered (Re $\simeq$ 230) is nearly 80%. In the case of body forces, the effect of electromagnetic forces on the drag coefficient is less than predicted. In this case the EBF not only accelerate the fluid in the neighborhood of the body, and thus increase the frictional drag, but in addition can increase the pressure in the wake and lower the pressure drag. Which of these two processes predominates depends on the nature of the variation of the drag coefficient under the action of EBF.

The present article is devoted to a numerical study of the flow around a sphere with internal sources of electromagnetic fields.

1. Suppose a viscous incompressible conducting fluid flows past a sphere of radius a; u_0 is the flow velocity at infinity; σ, ρ, and ν are respectively the conductivity, density, and kinematic viscosity of the fluid. Inside the sphere there is an electromagnetic field source which produces a traveling magnetic field in the surrounding fluid. Let the θ component of the field $\mathbf{H}$ on the outer boundary of the sphere in the spherical coordinate system (r, θ, α) be given in the form

$$H_\theta(1, \theta, t) = H_0 h_*(\theta)\, e^{-i(k\theta-\omega t)}, \quad 0 \leqslant \theta \leqslant \pi, \tag{1.1}$$

where r is the dimensionless radius obtained by using a as the unit of length, θ is the angle measured from the direction opposite $\mathbf{u}_0$, and H_0 is the maximum value of the field H_θ; the function $h_*(\theta)$ characterizes the distribution of the amplitude of the traveling wave over the surface of the sphere; $|h_*(\theta)|_{max} = 1$; ω is the frequency; k is the analog of the wave number, determining the number of half-waves between $\theta = 0$ and $\theta = \pi$ on the surface of the sphere; the ratio ω/k determines the phase angular velocity of the traveling wave. It is clear from (1.1) that the $\mathbf{E}$ and $\mathbf{H}$ fields do not depend on the α coordinate, and therefore an axisymmetric flow pattern is considered.

Internal currents in the sphere produce a traveling magnetic field, and the currents in the fluid exert a force on the sphere opposite $\mathbf{u}_0$. This force can cause the sphere to move, and therefore if H_0 is large enough the sphere is self-propelled. The problem is to determine the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ in the fluid, and to investigate the effect of EBF on the flow pattern and the magnitude of the drag coefficient for various values of H_0.

2. The electromagnetic fields in a fluid are generally determined together with the solution of the hydrodynamics equations. We consider the simple situation when the second

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term in Ohm's law $\mathbf{j} = \sigma[\mathbf{E} + (1/c)\mathbf{V} \times \mathbf{H}]$ can be neglected in comparison with the first, and thus Maxwell's equations are separated from the hydrodynamics equations. This is valid if the phase velocity of the traveling wave* is appreciably higher than the velocity of the fluid relative to the sphere, i.e.,

$$\omega a/k \gg u_0. \tag{2.1}$$

It should be noted that under condition (2.1) the electromagnetic system considered is energetically inefficient — the Joule losses far exceeding the useful mechanical work done by the electromagnetic body forces. However, in investigating the changes of hydrodynamic drag as a result of internal sources, this fact is not controlling.

The fields in the fluid are determined by the vector potential $\mathbf{A}(r, \theta, t) = H_0 a A(r, \theta) e^{i\omega t} \mathbf{e}_\alpha$, where the dimensionless function $A(r, \theta)$ satisfies the equation

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rA) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\cdot A)\right] - \frac{2i}{\delta^2}A = 0 \tag{2.2}$$

[$(\delta = c(a\sqrt{2\pi\sigma\omega})^{-1}$ is the dimensionless thickness of the skin layer] and the boundary conditions

$$\frac{\partial}{\partial r}[rA(r,\theta)]|_{r=1} = -h_*(\theta)e^{-ik\theta}, \quad A|_{r=\infty} = 0. \tag{2.3}$$

Expanding the boundary function in terms of associated Legendre functions $P_l^1(\cos\theta) = -\sin\theta(d/d\cos\theta)P_l(\cos\theta)$, i.e.,

$$h_*(\theta)e^{-ik\theta} = \sum_{l=1}^{\infty} d_l P_l^1(\cos\theta), \quad d_l = \frac{2l+1}{2l(l+1)}\int_{-1}^{1} h_*(\theta)e^{-ik\theta}P_l^1(\cos\theta)\,d\cos\theta,$$

we obtain the solution of problem (2.2), (2.3)

$$A(r,\theta) = \sum_{l=1}^{\infty} b_l \frac{1}{\sqrt{r}} H^{(2)}_{l+\frac{1}{2}}(sr)P_l^1(\cos\theta), \quad b_l = \frac{d_l}{lH^{(2)}_{l+\frac{1}{2}}(s) - sH^{(2)}_{l-\frac{1}{2}}(s)}, \tag{2.4}$$

where $s = (1 - i)/\delta$, and the $H^{(2)}_{l\pm 1/2}$ are Hankel functions of the second kind of half-integral order.

To calculate the total electromagnetic thrust on the sphere it is necessary to know the current distribution in the source producing the field (1.1), and the magnetic field produced over all space by the currents $\mathbf{j}$ in the fluid. Suppose the traveling magnetic field is produced by surface currents distributed over the sphere $r = 1$ with an α component

$$i_\alpha(\theta, t) = cH_0 i_*(\theta)e^{-i(k\theta-\omega t)}. \tag{2.5}$$

To determine the relation between the functions $i_*(\theta)$ and $h_*(\theta)$ we consider the fields $\mathbf{E}$ and $\mathbf{H}$ inside the sphere. The sphere is assumed nonconducting and nonmagnetic, and therefore the vector potential inside the sphere is described by the equation obtained from (2.2) by discarding the last term on the left-hand side. The solution of this equation which has no singularity at $r = 0$ has the form

$$A_1(r,\theta) = \sum_{l=1}^{\infty} b_l^{(1)} r^l P_l^1(\cos\theta). \tag{2.6}$$

From the boundary conditions

$$A(1,\theta) = A_1(1,\theta), \quad \frac{\partial}{\partial r}(rA_1)\Big|_{r=1} - \frac{\partial}{\partial r}(rA)\Big|_{r=1} = 4\pi i_*(\theta)e^{-ik\theta}$$

we obtain

$$b_l^{(1)} = b_l H^{(2)}_{l+\frac{1}{2}}(s);$$

*This can be said of a traveling wave when k is at least not less than unity. In the following hydrodynamics calculations it is assumed that k = 4.

$$4\pi i_*(\theta)\, e^{-ik\theta} = \sum_{l=1}^{\infty} b_l\left[(2l+1)\, H^{(2)}_{l+\frac{1}{2}}(s) - sH^{(2)}_{l-\frac{1}{2}}(s)\right] P_l^1(\cos\theta). \tag{2.7}$$

Thus, the electric and magnetic fields in the fluid and inside the sphere are described by solutions (2.4) and (2.6), respectively (quantities referring to the space inside the sphere from now on are denoted by the subscript 1), and the distribution of surface current is given by Eqs. (2.5) and (2.7). The magnetic field produced by the currents in the fluid is determined by the differences $A(r, \theta) - A^0(r, \theta)$ and $A_1(r, \theta) - A_1^0(r, \theta)$, where $A_1^0(r, \theta) = \sum_{l=1}^{\infty} c_l r^l P_l^1(\cos\theta)$, $A^0(r,\theta) = \sum_{l=1}^{\infty} c_l r^{-(l+1)} P_l^1(\cos\theta)$ describe the fields produced by the current (2.4) in empty space, where

$$c_l = \frac{b_l}{2l+1}\left[(2l+1)\, H^{(2)}_{l+\frac{1}{2}}(s) - sH^{(2)}_{l-\frac{1}{2}}(s)\right].$$

3. The electromagnetic body forces $\mathbf{f}$ and the curl of these forces under condition (2.1) are also calculated independently of the velocity field. Since $\mathbf{f} = (\sigma/c)[\mathbf{E}\times\mathbf{H}]$, $\mathbf{E} = -(1/c)\partial A/\partial t = -(i\omega/c)H_0 a A(r,\theta)e^{i\omega t}\,\mathbf{e}_\alpha$, $\mathbf{H} = \operatorname{rot}\mathbf{A}$, the result reduces to the form

$$f_r = -f_0\frac{1}{r}\operatorname{Real}\left[iA\frac{\partial}{\partial r}(rA)\, e^{2i\omega t} - iA^*\frac{\partial}{\partial r}(rA)\right], \tag{3.1}$$

$$f_\theta = f_0\frac{1}{r\sin\theta}\operatorname{Real}\left[-iA\frac{\partial}{\partial\theta}(\sin\theta\cdot A)\, e^{2i\omega t} + iA^*\frac{\partial}{\partial\theta}(\sin\theta\cdot A)\right],\quad f_\alpha = 0;$$

$$\operatorname{rot}\mathbf{f} = (f_0/a)[\Phi(r,\theta) + \Omega(r,\theta,t)]\mathbf{e}_\alpha, \tag{3.2}$$

$$\Phi(r,\theta) = \frac{2}{r}\operatorname{Real}\left(i\frac{\partial A^*}{\partial r}\frac{\partial A}{\partial\theta}\right),\quad \Omega(r,\theta,t) = \frac{1}{r}\operatorname{Real}\left[2iA\left(\frac{1}{r}\frac{\partial A}{\partial\theta} - \frac{\cos\theta}{\sin\theta}\frac{\partial A}{\partial r}\right)e^{2i\omega t}\right],$$

where $f_0 = \sigma\omega H_0^2 a/2c^2$ is the scale of the body force density, and the function $A(r, \theta)$ is given by Eq. (2.4). It is clear from (3.1) and (3.2) that the force $\mathbf{f}$ has both a stationary component and a component which oscillates with a frequency 2ω. The electromagnetic thrust $\mathbf{F}$ on the sphere in the direction opposite $\mathbf{u}_0$ consists of two analogous parts. The time average of the magnitude of this force can be written in the form

$$\langle F\rangle = f_0 a^3 F_1(k, \delta). \tag{3.3}$$

In calculating the dimensionless force $F_1(k, \delta)$ we bear in mind the fact that $\mathbf{F}$ represents the vector sum of the forces acting on the surface currents (2.5) due to the magnetic field produced by currents $\mathbf{j}$ in the fluid. Hence it follows that

$$F_1(k,\delta) = 8\pi^2\delta^2\operatorname{Real}\int_0^\pi \sin\theta\cdot i_*(\theta)\, e^{-ik\theta}\,[h_{j\theta}(1,\theta)\cos\theta + h_{jr}(1,\theta)\sin\theta]^* d\theta, \tag{3.4}$$

$$h_{j\theta}(r,\theta) = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(A_1 - A_1^0\right)\right],\quad h_{jr}(r,\theta) = \frac{\partial}{r\sin\theta\partial\theta}\left[\sin\theta\cdot\left(A_1 - A_1^0\right)\right].$$

On the other hand $\mathbf{F}$ is a vector opposite the resultant of the EBF in the whole fluid [therefore as the scale of the force (3.3) we take the product of the scale of f_0 times the characteristic volume], and consequently

$$F_1(k,\delta) = -\int_1^\infty\int_0^\pi \frac{1}{f_0}\left(\langle f_r\rangle\cos\theta - \langle f_\theta\rangle\sin\theta\right) 2\pi r^2\sin\theta\, d\theta\, dr.$$

Calculations were performed with Eq. (3.4), and the expression in terms of the volume integral was used to check the calculations. The results for $h_*(\theta) = \sin\theta$ and $\delta = 2$ are listed in Table 1. Using the parameter of the MHD interaction

$$N = \frac{1}{4}\frac{f_0 a}{\rho u_0^2/2} = \frac{\sigma\omega a^2 H_0^2}{4c^2\rho u_0^2}, \tag{3.5}$$

characterizing the ratio of the work done by the body forces to the dynamic pressure, we write Eq. (3.3) in the form

TABLE 1

k	1,0	1,5	2,0	3,0	4,0	5,0	6,0	7,0
$F_1(k, 2)$	0,792	0,950	0,932	0,620	0,334	0,196	0,131	0,094

$$\langle F\rangle = (4/\pi)\, N\pi a^2 \left(\rho u_0^2/2\right) F_1(k, \delta). \tag{3.6}$$

4. The investigation of the flow pattern is reduced to the solution of the hydrodynamics equations

$$\operatorname{div} \mathbf{V} = 0,\ \operatorname{rot} \mathbf{V} = \mathbf{W}, \qquad (4.1)$$
$$\partial \mathbf{W}/\partial t - \operatorname{rot}\,[\mathbf{V} \times \mathbf{W}] + \nu \operatorname{rot} \operatorname{rot} \mathbf{W} = (1/\rho) \operatorname{rot} \mathbf{f}$$

with body forces whose curl is given in (3.2). The presence of the oscillating term $\Omega(r, \theta, t)$ in the curl of the forces leads to the superposition of a small [under condition (2.1)] nonstationary addition on the stationary flow pattern.* We are interested in the time average of the velocity and pressure distributions, which are calculated without taking account of the effect of Reynolds stresses resulting from the variable velocity component. In this case the system of equations (4.1) is equivalent to the equations

$$E^2\psi - rw \sin\theta = 0; \tag{4.2}$$

$$-\frac{1}{2}\frac{1}{r}\left[\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\left(\frac{w}{r\sin\theta}\right) - \frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}\left(\frac{w}{r\sin\theta}\right)\right] + \frac{1}{\mathrm{Re}}\frac{1}{r\sin\theta}E^2(r\sin\theta\cdot w) + N\Phi(r,\theta) = 0 \tag{4.3}$$

for the dimensionless stream function $\psi(r, \theta)$ and the vorticity $w(r, \theta)$, introduced by the relations

$$\mathbf{V} = u_0\mathbf{v},\ \mathbf{v} = \frac{1}{r\sin\theta}\left[-\frac{1}{r}\frac{\partial\psi}{\partial\theta}\mathbf{e}_r + \frac{\partial\psi}{\partial r}\mathbf{e}_\theta\right], \quad \mathbf{W} = \frac{u_0}{a} w(r,\theta)\,\mathbf{e}_\alpha.$$

Here $\mathrm{Re} = u_0 2a/\nu$ is the Reynolds number, the parameter N is defined in (3.5), and the operator $E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$. By using the velocity field obtained, the pressure distribution P is determined from the equation of motion

$$(\mathbf{V}\nabla)\mathbf{V} = -(1/\rho)\nabla P + \nu\Delta\mathbf{V} + (1/\rho)\mathbf{f}.$$

The result for the distribution of the dimensionless pressure $p = P/(\rho u_0^2/2)$ over the surface of the sphere takes the form

$$p(\theta) = p_0 + \frac{4}{\mathrm{Re}}\int_0^\theta \frac{\partial}{\partial r}(rw)\Big|_{r=1} d\theta + 4N\frac{1}{f_0}\int_0^\theta \langle f_\theta(1,\theta)\rangle\, d\theta, \tag{4.4}$$
$$p_0 = 1 + \frac{8}{\mathrm{Re}}\int_1^\infty \frac{1}{r}\frac{\partial w}{\partial\theta}\Big|_{\theta=0} dr,$$

where p_0 is the pressure at the front critical point $r = 1$, $\theta = 0$ (the pressure at infinity is taken equal to zero); $(1/f_0)\langle f_\theta(1, \theta)\rangle$ is the time average of the dimensionless θ component of the body forces on the surface of the sphere. According to (3.1), $(1/f_0)\,\langle f_\theta(r,\theta)\rangle = \frac{1}{r\sin\theta}\,\mathrm{Real}\left[iA^*\frac{\partial}{\partial\theta}(\sin\theta\cdot A)\right]$.

5. The equations derived were solved numerically by the method used in (4), in which a new independent variable $z = \ln r$ is used instead of r. In the variables (z, θ) Eqs. (4.2) and (4.3) take the form

*The ratio of the scale of the oscillating component of the vorticity to the scale of its stationary component is determined by the ratio of two characteristic times: the period $1/\omega$ of the oscillating part $\Omega(r, \theta, t)$ and the residence time a/u_0 of a fluid particle in the force field. This ratio is equal to $u_0/a\omega$, and on the basis of (2.1) is substantially smaller than unity.

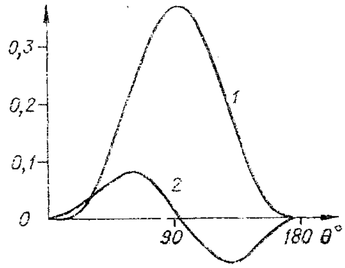

Fig. 1

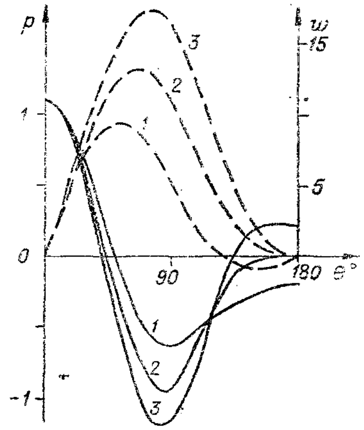

Fig. 2

$$L\psi - ge^{2z} = 0,$$

$$\frac{\partial w}{\partial z} + \frac{\mathrm{Re}}{2} e^{-2z}\left(\frac{\partial\psi}{\partial z}\frac{\partial\chi}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial\chi}{\partial z}\right) - \frac{1}{\sin\theta} e^{-3z} Lq - \mathrm{Re}\, N\Phi\left(e^{z}, \theta\right) = 0,$$

where $L = \partial^2/\partial z^2 - \partial/\partial z + \partial^2/\partial\theta^2 - \cot\theta(\partial/\partial\theta)$; q and χ are auxiliary quantities related to the vorticity:

$$q = wr\sin\theta,\ \chi = (1/r\sin\theta)w.$$

The domain of definition of the required functions is the half-strip $0 \leqslant \theta \leqslant \pi$, $z \geqslant 0$, which is replaced by a rectangle with the boundary $z = z_m$ for the numerical solution. A net uniform with respect to both $z(\Delta z = z_m/N_1)$ and $\theta(\Delta\theta = \pi/N_2)$ is used in the (z, θ) plane. The uniform division of the interval $0 \leqslant z \leqslant z_m$ corresponds to a nonuniform net in the physical (r, θ) plane, finest close to the surface of the sphere where the flow is characterized by maximum gradients, and coarser far from the sphere.

At the boundaries $\theta = 0$, $\theta = \pi$, and $z = z_m$ the following boundary conditions were used:

$$\psi = 0,\ w = 0 \quad \text{at} \quad \theta = 0,\ \theta = \pi;$$

$$w = 0,\ \psi = (1/2)e^{2z}\sin^2\theta \quad \text{at} \quad z = z_m,\ 0 \leqslant \theta \leqslant \pi/2;$$

$$w = 0,\ \partial\psi/\partial x = 0 \quad \text{at} \quad z = z_m,\ \pi/2 < \theta < \pi,$$

with the x axis along the axis of symmetry. The condition $\psi = 0$ and the boundary condition of [5] with the relaxation procedure of [6] for vorticity were used on the solid boundary.

The calculations were performed for a net with $N_1 = 70$ and $N_2 = 50$; the outer boundary of the region was assigned the value $z_m = 2$, representing a spherical surface with a radius 7.4 times as large as the radius of the solid sphere. The errors resulting from the relative coarseness of the net and the proximity of the outer boundary were estimated from a comparison of the results obtained for no electromagnetic fields and Re = 100 with data of [7] obtained with $N_1 = 100$, $N_2 = 60$, and $z_m = 2.5$. The error in the magnitude of the vorticity on the surface of the sphere did not exceed 1%. The largest error occurred in the calculation of the pressure distribution in the separation region; the maximum relative error (ratio of the absolute error to the pressure at the frontal point) was 1.6% at the point $\theta = \pi$, $r = 1$. In view of this, the error in the pressure drag coefficient was also

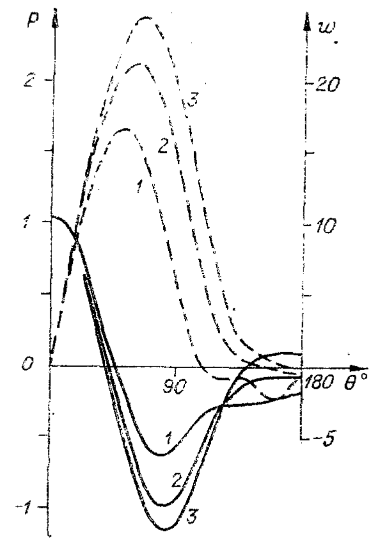

Fig. 3

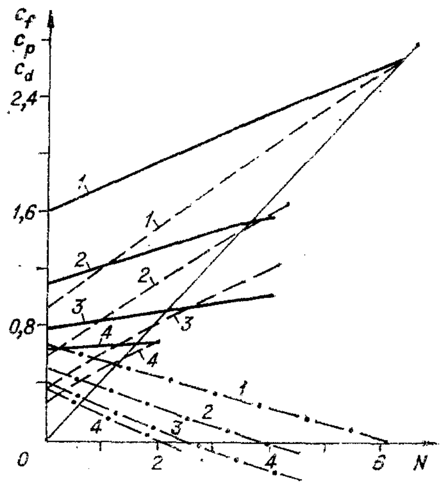

Fig. 4

appreciable, amounting to 1.8%. The error in the calculation of the frictional drag coefficient was 0.5%, and the relative error of the total drag coefficient was 0.8%. The error in calculating the total drag coefficient for Re = 300 was 1.3% when the outer boundary was approximated up to z_m = 1.5 with a decrease in the mesh size in z.

6. The calculations were performed for δ = 2 and k = 4 for the function $h_*(\theta)$ = sin θ, in which the amplitude of the traveling wave was maximum in the equatorial plane. The distributions of the dimensionless θ and r components of the body forces over the surface of the sphere obtained in this way are shown in Fig. 1. Here curve 1 represents $(1/f_0)\langle f_\theta(1,\ \theta)\rangle$, and curve 2 $(1/f_0)\langle f_r(1, \theta)\rangle$. It is clear that the forces **f** have basically the direction (coinciding with $\mathbf{e}_\theta$) which intuitively seems necessary to prevent flow separation.

Figure 2 shows the pressure distribution (solid curves) and vorticity (open curves) over the surface of the sphere for Re = 100 and N = 0, 2, and 4 (curves 1-3, respectively). Similar relations for Re = 300 are shown in Fig. 3 (curves 1-3 correspond to N = 0, 1, and 1.75). It is clear that the presence of EBF has little effect on the flow parameters near the frontal point of the sphere, and has the greatest effect in the wake. With an increase in N the separation point is shifted back and disappears, and the pressure in the wake increases, leading to a decrease of the pressure drag. The vorticity w on the surface of the sphere increases with increasing N, increasing the frictional drag. Figure 4 shows the dependence of the frictional c_f (dashed curves) and pressure c_p (dash–dot curves) drag coefficients and the total drag coefficient $c_d = c_f + c_p$ (solid curves) on N for all values of Re considered. The numbers 1-4 denote curves referring respectively to Re = 50, 100, 200, and 300. It is clear that c_d increases most rapidly with increasing N for Re = 50. This growth slows down with increasing Re, and for Re = 300 the drag coefficient is practically independent of N. The similar behavior of the dependence of c_d on N is accounted for by the decrease of the relative contribution of the frictional drag to the total drag with increasing Reynolds number.

The value $N = N_*$ at which the sphere becomes self-propelled is determined by equating the thrust (3.6) to the resistance force $\pi a^2 (\rho u_0^2/2) c_d(N)$, i.e., from the condition

$$c_d(N) = (4/\pi) F_1(k, \delta) N.$$

The intersection of the thin straight line of Fig. 4, which is a plot of $y(N) = (4/\pi) \cdot F_1(4, 2) N$, with the $c_d(N)$ curve determines the required values of N_*. For Re = 50, $N_* \simeq 6.7$ and $c_d(N_*) \simeq 2.85$; i.e., the drag on a self-propelled sphere is 1.78 times that of classical flow. For Re = 100, 200, and 300 the values under consideration are, respectively, $N_* \simeq 3.6$, $c_d(N_*) \simeq 1.52$; $N_* \simeq 2.1$, $c_d(N_*) = 0.9$; and $N_* \simeq 1.6$, $c_d(N_*) \simeq 0.68$. Thus, even for Re = 300 the drag coefficient of a sphere set in motion by the electromagnetic forces under consideration is only slightly larger than the value $c_d(0) = 0.64$, the drag coefficient for classical flow around a sphere. This fact gives us confidence that for larger values of Re the drag for MHD flow around a body may be smaller than for classical flow, as predicted by Merkulov [8]. It should be emphasized that for this to be the case the electric and magnetic fields must be produced by a source near (inside) the body around which the flow occurs. For flow around a body in an external magnetic field the presence of the field always leads to an increase in both the pressure drag and the total hydrodynamic drag [1, 2].

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